Basic financial calculations

Session 2

Overview

- A few finance basics and their Excel implementations
  - Net present value (NPV)
  - Internal rate of return (IRR)
  - Payment schedules and loan tables
  - Future value
  - Compounded interest

Summary

1. Present Value and Net Present Value
2. Internal Rate of Return and Loan Tables
3. Multiple Internal Rates of Return
4. Flat Payment Schedules
5. Future Values and Applications
6. A Pension Problem – Complicating the Future-Value Problem
7. Continuous Compounding
8. Discounting Using Dated Cash Flows

Basic Financial Calculations

1. PRESENT VALUE AND NET PRESENT VALUE

1. Present value and net present value

- Both concepts are related to the value *today* of a set of future anticipated cash flows.

- *Example: we are valuating an investment that promises $100 per year at the end of this and the next four years. We suppose that there is no doubt that this series of 5 payments of $100 each will actually be paid. If a bank pays an annual interest rate of 10% on a 5 year deposit, then this 10% is the investiment's opportunity cost, the alternative benchmark return to which we want to compare the investment. We may calculate the value of the investment by discounting its cash flows using this opportunity cost as a discount rate.*

1. Present value and net present value

- The *present value* of a series of cash flows is the value today of the cash flows starting in year 1

$$\text{Present value} = \sum_{t=1}^{N} \frac{CF^t}{(1+r)^t}$$

- The *net present value* is the present value and **the cost of acquiring** the asset (the cash flow at time zero)

$$\text{Net present value} = \sum_{t=0}^{N} \frac{CF^t}{(1+r)^t} = CF_0 + \sum_{t=1}^{N} \frac{CF^t}{(1+r)^t}$$

1. Present value and net present value

- Computing the present value (equal cash flows)

Feuille de calcul

- Computing the present value (non equal cash flows)

Feuille de calcul

1. Present value and net present value

- Net present value
  - Using the same example, suppose that the series of 5 cash flows of $100 is sold for $250. What would be the NPV?

Feuille de calcul

1. Present value and net present value

- Present value of an annuity
  - Def.: an *annuity* is a security that pays a constant sum in each period in the future. Annuities may have a finite or infinite series of payments. If the annuity is finite and the appropriate discount rate if *r,* then the value today of the annuity is its present value.
  - Def.: a *growing annuity* pays out a sum C that grows at a periodic growth rate *g.*

1. Present value and net present value

- PV of finite annuity $= \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^n} = C\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right)$

Feuille de calcul

- PV of infinite annuity $= \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots = \frac{C}{r}$

Feuille de calcul

- PV of finite growing annuity

Feuille de calcul

$$= \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots + \frac{C(1+g)^{n-1}}{(1+r)^n} = \frac{C\left[1 - \left(\frac{1+g}{1+r}\right)^n\right]}{r-g}$$

- PV of infinite growing annuity

Feuille de calcul

$$= \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots = \frac{C}{r-g},\ provided \left|\frac{1+g}{1+r}\right| < 1$$

Basic Financial Calculations

2. INTERNAL RATE OF RETURN AND LOAN TABLES

2. IRR and loan tables

- The Internal Rate of Return (IRR) is defined as the compound rate of retun *r* that makes the NPV equal to zero:

$$CF_0 + \sum_{t=1}^{N} \frac{CF^t}{(1+r)^t} = 0$$

Feuille de calcul

- Determining the IRR by trial and error using Excel's Goal Seek

Feuille de calcul

2. IRR and loan tables

- The IRR is the compound *rate of return paid by the investment.* It helps to make a *loan table*, which shows the division of the investment's cash flows between investment income and the return of the investment principal.

Feuille de calcul

- It could be access by the IRR function of excel, with the loan tables (goal seek tool), or with Excel's rate function

Feuille de calcul

Basic Financial Calculations

3. MULTIPLE INTERNAL RATES OF RETURN

3. Multiple internal rates of return

- Sometimes a series of cash flows has more than one IRR

- Multiple internal rates of return

Feuille de calcul

- Bond cash flows: NPV crosses x-axis only once, so there is only one IRR

Feuille de calcul

Basic Financial Calculations

4. FLAT PAYMENT SCHEDULES

4. Flat payments schedules

- Another common problem is to compute a flat repayment for a loan.

- *Example: you take a loan for $10000 at an interest rate of 7% per year. The bank wants you to make a series of payments that will pay off the loan and the interest over 6 years.* Using Excel's PMT function, it is possible to determine how much each annual payment should be.

Feuille de calcul

Basic Financial Calculations

5. FUTURE VALUES AND APPLICATIONS

5. Future values and applications

- The *future value* of $10000 in 10 years at 10% per year is:

$$FV = 1000 * (1 + 10\%)^{10} = 2593,74$$

- If you take into account the deposit of the interests accumulated each year, the *future value* is:

$$FV = \sum_{t=1}^{10} 1000 * (1 + 10\%)^{t}$$

Feuille de calcul

Basic Financial Calculations

6. A PENSION PROBLEM – COMPLICATING THE FUTURE-VALUE PROBLEM

6. A pension problem

- You are currently 55 years old and intend to retire at age 60. To make your retirement easier, you intend to start a retirement account
  - At the beginning of each oh years 1,2,3,4, you intend to make a deposit into the retirement account that will earn 8% per year.
  - After retirement at 60 years old, you anticipate to live 8 more years. At the beginning of each year you intend to withdraw 30000$. The account balance continue to earn 8%.
- How much should you deposit annually in the account?

6. A pension problem

- Solving the problem using Excel's solver (in the Tools menu)

Feuille de calcul

- Solving the problem using financial formulas

$$\sum_{t=0}^{4} \frac{Initial\ deposit}{(1,08)^{4}} - \sum_{t=5}^{12} \frac{30000}{(1,08)^{t}}$$

$$Initial\ deposit = [\sum_{t=5}^{12} \frac{30000}{(1,08)^{t}}] / [\sum_{t=0}^{4} \frac{1}{(1,08)^{t}}]$$

Feuille de calcul

Basic Financial Calculations

7.CONTINUOUS COMPOUNDING

7.Continuous compounding

- You are deposit 1000$ in a bank account that pays 5% per year. At the end of the year you will have $1000*(1,05)=$1050. Now suppose that the bank interprets « 5% per year » to mean that it pays you 2,5 percent interest twice a year. Thus after six months you'll have $1025, and after one year you will have $1050,625.

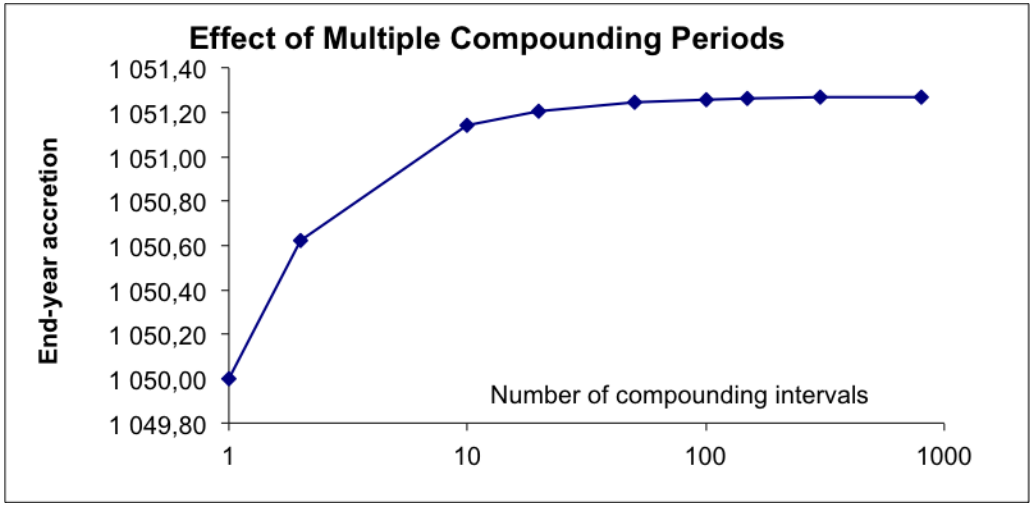
- If you get interest *n* times a year, your accretion at the end of the year is

$$\$1000 * (1 + \frac{0,05}{n})^n$$

7.Continuous compounding

- As *n* increases, this amount gets larger, converging to $e^{0,05}$. When *n* is infinite, it is called *continuous compounding.*

- Multiple compounding periods




Feuille de calcul

7.Continuous compounding

- If the accretion factor for continuous compounding at interest ***r*** over ***t*** years is e^{rt}, then the discount factor for the same period is e^{-rt}. Thus a cash flow C_t occurring in year ***t*** and discounted at continuously compounded rate ***r*** will be worth $C_t e^{-rt}$ today.

Feuille de calcul

- Calculating the continuously compounded return from price data depends on the compounding method.

Feuille de calcul

Basic Financial Calculations

8. DISCOUNTING USING DATED CASH FLOWS

Suppose at time 0 you had \$1,000 in the bank and suppose that one year later you had \$1,200. What was your percentage return? Although the answer may appear obvious, it actually depends on the compounding method. If the bank paid interest only once a year, then the return would be 20 percent:

$$\frac{1{,}200}{1{,}000} - 1 = 20\%$$

However, if the bank paid interest twice a year, you would need to solve the following equation to calculate the return:

$$1{,}000 * \left(1 + \frac{r}{2}\right)^2 = 1{,}200 \Rightarrow \frac{r}{2} = \left(\frac{1{,}200}{1{,}000}\right)^{1/2} - 1 = 9.5445\%$$

The annual percentage return when interest is paid twice a year is therefore $2 * 9.5445\% = 19.089\%$.

In general, if there are n compounding periods per year, you have to solve $\frac{r}{n} = \left(\frac{1{,}200}{1{,}000}\right)^{1/n} - 1$ and then multiply the result appropriately. If n is very large, this converges to $r = \ln\left(\frac{1{,}200}{1{,}000}\right) = 18.2322\%$:

8. Discounting using dated cash flows

- With the previous examples, cash flows occured at fixed periodic intervals
- Two Excel functions allow to do computations on cash flows which occur on specific dates that are not at even intervals *(NOTE: if you do not see these functions, add them in by going to Tools/Add-ins, on the tool bar and checking Analysis ToolPak)*.
  - Using XIRR to compute the annualized IRR

    Feuille de calcul

  - Using XNPV to compute the NPV

    Feuille de calcul