Options: the mechanics

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Calls and Puts

- A call gives the holder the right to buy an asset at a certain price within a specific period of time. Calls are similar to having a **long position** on a stock. Buyers of calls hope that the stock will increase substantially before the option expires.

- A put gives the holder the right to sell an asset at a certain price within a specific period of time. Puts are very similar to having a **short position** on a stock. Buyers of puts hope that the price of the stock will fall before the option expires.

- The price at which an underlying stock can be purchased or sold is called the strike price.
  - This is the price a stock price must go above (for calls) or go below (for puts) before a position can be exercised for a profit. All of this must occur before the expiration date.

- An option that is traded on a national options exchange such as the Chicago Board Options Exchange (CBOE) is known as a **listed option**.
- These have **fixed strike prices** and **expiration dates**.
- Each listed option represents 100 shares of company stock (known as a **contract**).

- For call options, the option is said to be **in-the-money** if the share price is above the strike price.
- A put option is **in-the-money** when the share price is below the strike price.
- The amount by which an option is **in-the-money** is referred to as **intrinsic value**.

- The total cost (the price) of an option is called the **premium**.
- This price is determined by factors including the stock price, strike price, time remaining until expiration (time value) and volatility.

Example

- Let's say that on May 1, the stock price of Cory's Tequila Co. is $67 and the premium (cost) is $3.15 for a July 70 Call, which indicates that the expiration is the third Friday of July and the strike price is $70.
- The total price of the contract is $3.15 x 100 = $315. In reality, you'd also have to take commissions into account, but we'll ignore them for this example.

Example

- Remember, a stock option contract is the option to buy 100 shares; that's why you must multiply the contract by 100 to get the total price.
- The strike price of $70 means that the stock price must rise above $70 before the call option is worth anything; furthermore, because the contract is $3.15 per share, the break-even price would be $73.15.

Example

- When the stock price is $67, it's less than the $70 strike price, so the option is worthless.
- But don't forget that you've paid $315 for the option, so you are currently down by this amount.

Example

- Three weeks later the stock price is $78.
  - The options contract has increased along with the stock price and is now worth $8.25 x 100 = $825.
  - Subtract what you paid for the contract, and your profit is ($8.25 - $3.15) x 100 = $510. You almost doubled our money in just three weeks!
- You could sell your options, which is called "closing your position," and take your profits - unless, of course, you think the stock price will continue to rise. For the sake of this example, let's say we let it ride.

Example

- By the expiration date, the price drops to $62.
  - Because this is less than our $70 strike price and there is no time left, the option contract is worthless.
  - We are now down to the original investment of $315.

Example

- To recap, here is what happened to our option investment:

Date	May 1	May 21	Expiry Date
Stock Price	$67	$78	$62
Option Price	$3.15	$8.25	worthless
Contract Value	$315	$825	$0
Paper Gain/Loss	$0	$510	-$315

- The price swing for the length of this contract from high to low was $825, which would have given us over double our original investment.
- This is leverage in action!

American vs European options

- European option gives right to buy/sell stock **only on** the exercise date T
- American option gives the right to buy/sell stock **on or before** exercise date T

Notation

- C for call
  - Time subscript if necessary: C_0 or C_t
- P, $P_{0,}$ P_t for put
- X or K for exercise price
  - Also called *strike price*
- S, $S_{0,}$ S_t for stock price
- r for interest rate
- σ for standard deviation of stock return

- Intrinsic value of a call:
  - $C = \max(S - X, 0)$
- Intrinsic value of a put:
  - $P = \max(X - S, 0)$

Call option cash flows

Call Option Payoff Patterns

Time 0		Time T	
Purchase call option, cash flow < 0	Between times 0 and T: Cash flow = 0 for European option; Cash flow $\geq$ 0 for American option	Terminal call payoff, Max[S_T - X,0] $\geq$ 0	Cash flows of call buyer
Write (I.e., issue) call option, cash flow > 0	Between times 0 and T: Cash flow = 0 for European option; Cash flow $\leq$ 0 for American option	Pay terminal call payoff = - Max[S_T - X, 0] $\leq$ 0	Cash flows of call writer

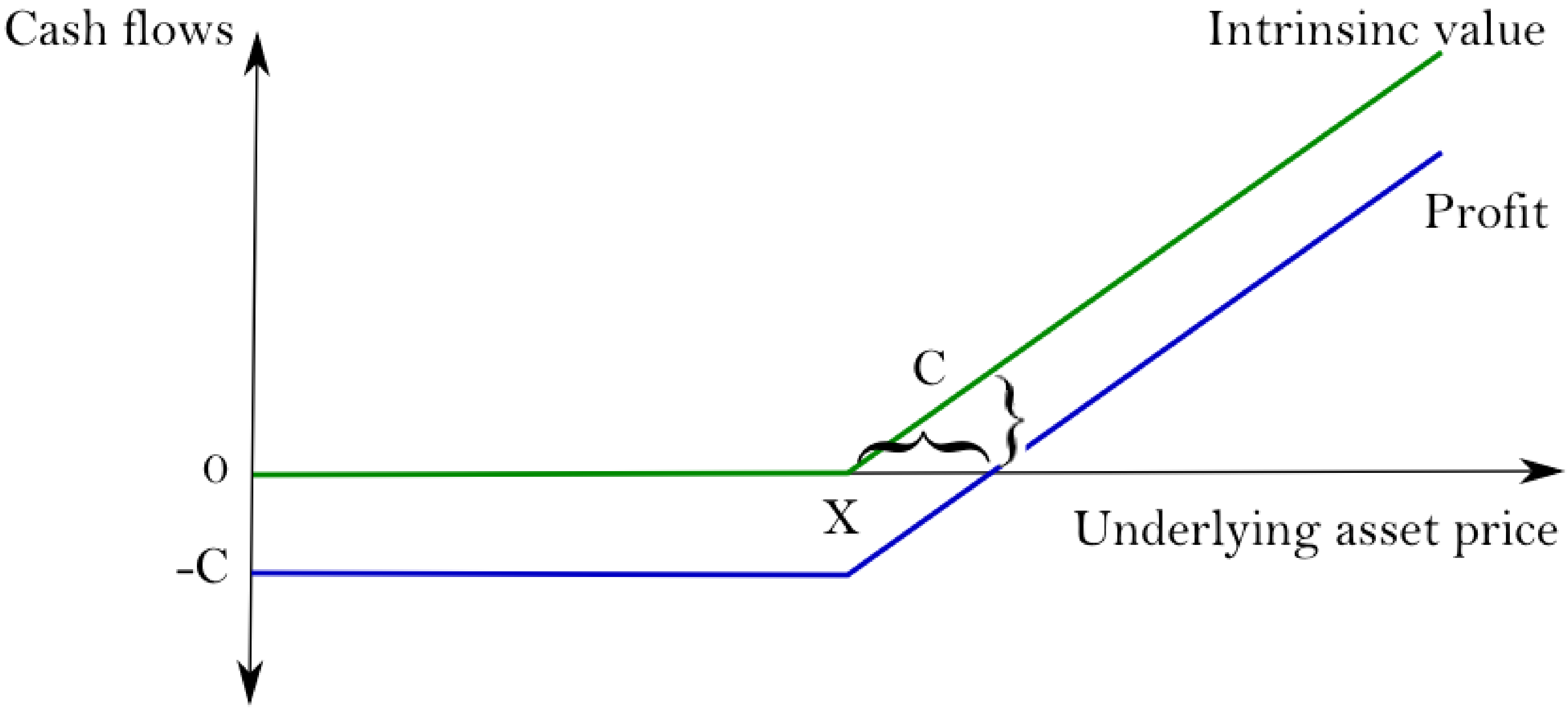
Option buyers and option writers

- Options can be bought or sold
- Like long or short positions in stock or other assets
- **Call option buyer**
  - Pays money up front, gets money if call ends **in the money**, i.e. if $S_T - X > 0$
- **Call option writer/seller**
  - Gets money up front, pays money if call ends **in the money**, i.e. if $S_T - X > 0$

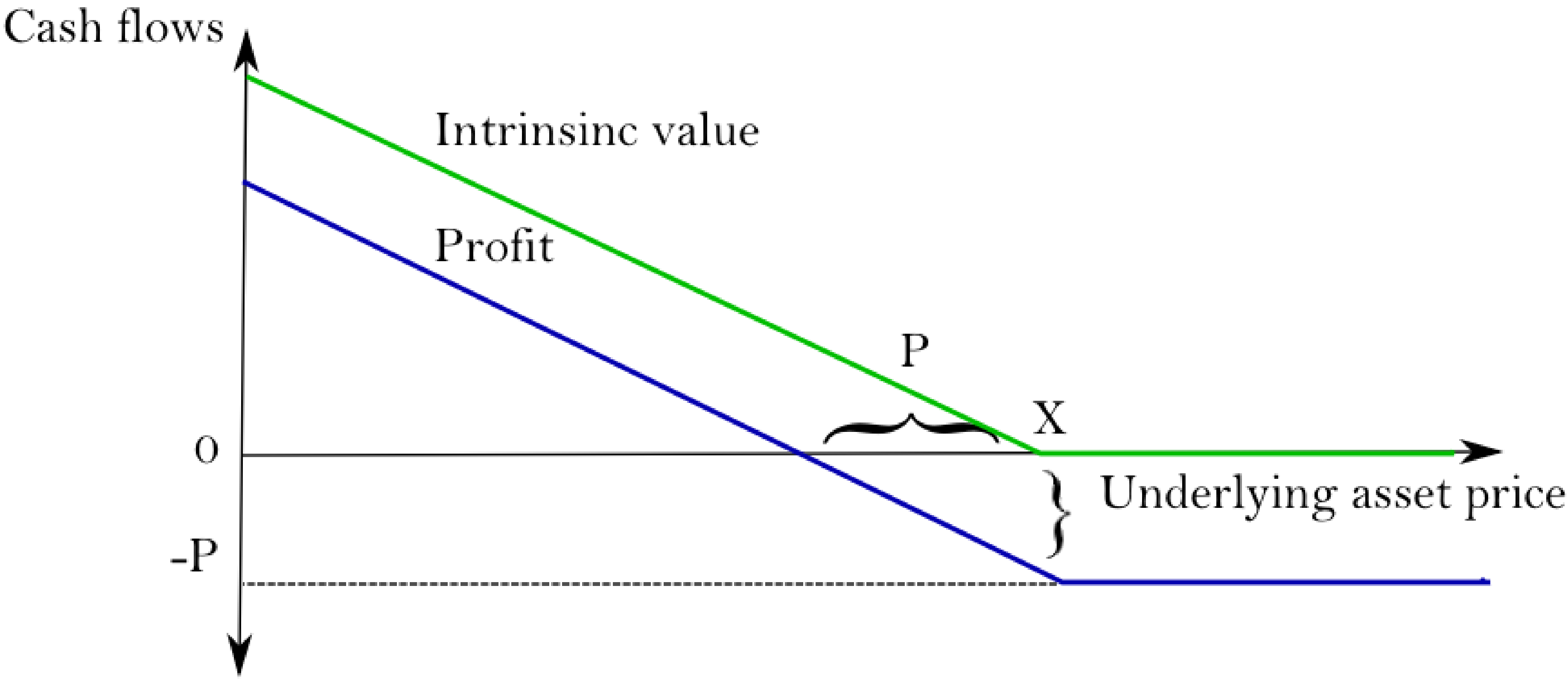
Put option cash flows

Put Option Payoff Patterns

Time 0		Time T	
Purchase put option, cash flow < 0	Between times 0 and T: Cash flow = 0 for European option Cash flow $\geq$ 0 for American option	Terminal put payoff, Max[X - S_T,0] $\geq$ 0	Cash flows of put buyer
Write (I.e., issue) call option, cash flow > 0	Between times 0 and T: Cash flow = 0 for European option Cash flow $\leq$ 0 for American option	Pay terminal call payoff = - Max[X - S_T, 0] $\leq$ 0	Cash flows of put writer

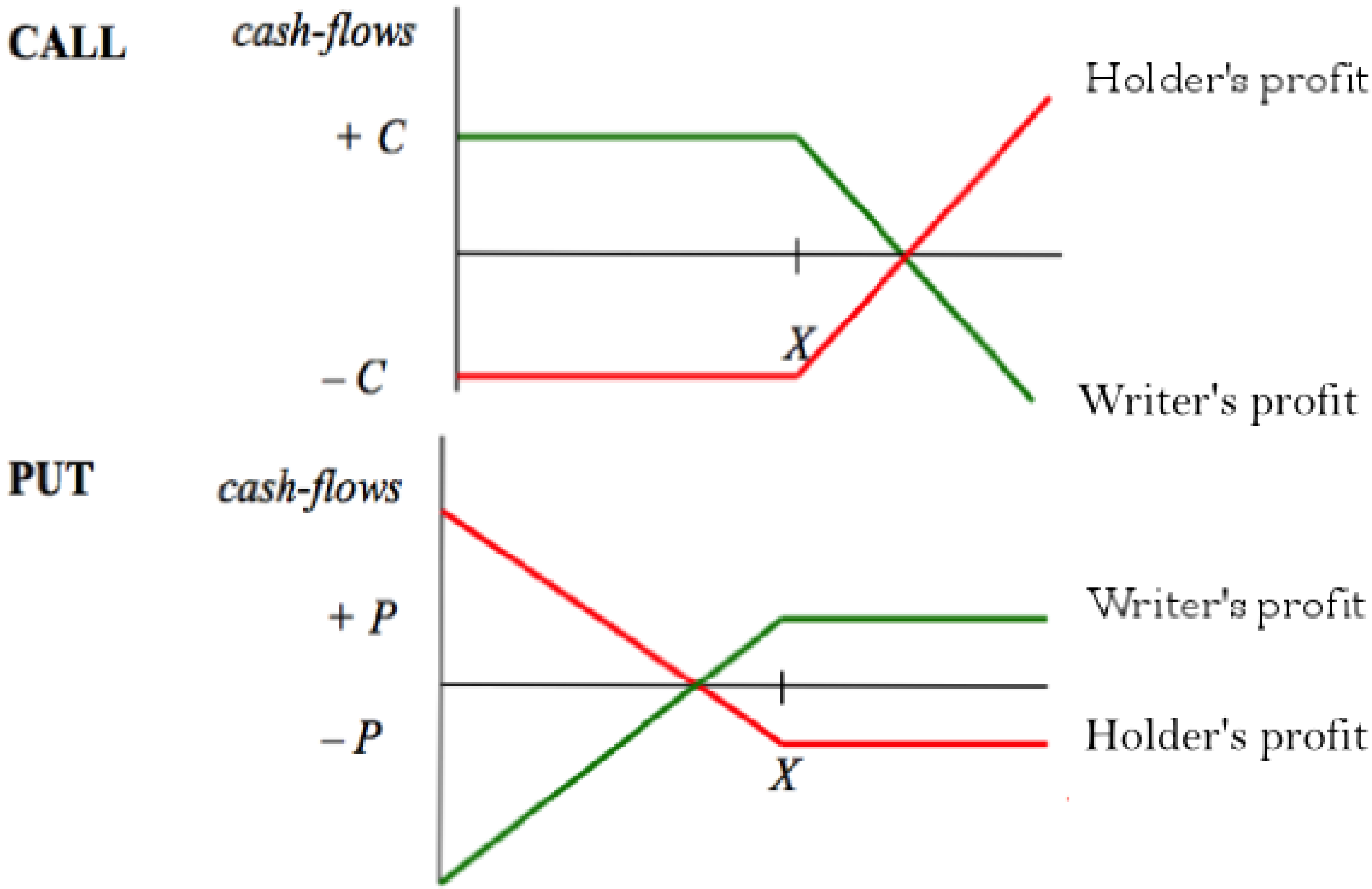

Cash flows
Intrinsinc value
Profit
C
0
X
-C
Underlying asset price

Call	**Buyer**		**Seller**	
Premium	-C		+C	
At maturity:	S<X	S>X	S<X	S>X
Decision	no	yes	n.a.	n.a.
Intrinsic value	0	S-X	0	-(S-X)
Profit	-C	S-X-C	+C	-(S-X)+C



Cash flows
Intrinsinc value
Profit
P
X
0
-P
Underlying asset price

Put	Buyer		Seller	
Premium	-P		+P	
At maturity:	S<X	S>X	S<X	S>X
Decision	yes	no	n.a.	n.a.
Intrinsic value	X-S	0	-(X-S)	0
Profit	X-S-P	-P	-(X-S)+P	+P



CALL
cash-flows
+ C
– C
X
Holder's profit
Writer's profit
PUT
cash-flows
+ P
– P
X
Writer's profit
Holder's profit

How to think about options

- Call is a *bet that the stock price will go* ***up*** in the future
  - Call is a way *to delay the purchase of stock*
    - Instead of buying stock, buy the right to buy the stock in the future (the call)
- Put is a *bet that the stock price will go* ***down*** *in the future*
  - Put is a way *to delay the sale of stock*
    - Instead of selling the stock, buy the right to sell the stock in the future (the put)

Option is a one-sided bet

- When you buy a call or put, you can
  - Make an unlimited amount of money in the future
  - You can only lose your original investment
- Finance is full of two-sided bets
  - Stocks: If price goes up/down by \$10, you gain/lose \$10
  - Futures contract: Contract to buy commodity in future for a given price. Losses/gains are symmetric

Really general property of calls

- The higher the exercise price, the lower the price of the call
  - Call with X = 40 is a bet that the stock price will be > 40 in the future
  - Call with X = 50 is a bet that the stock price will be > 50 in the future (less likely, hence less valuable)

Really general property of puts

- The higher the exercise price, the higher the price of the put
  - Put with X = 40 is a bet that the stock price will be < 40 in the future
  - Put with X = 50 is a bet that the stock price will be < 50 in the future (more likely, hence more valuable)

Really general property of calls and puts

- Longer-maturity puts and calls are more valuable
  - Call with X = 40 and maturity T = 6 months
  - Call with X = 40 and maturity T = 1 year
- Interpretation
  - More time to see stock price rise > 40
  - More time to delay purchase of stock

A more difficult property

- Options are only interesting when the underlying stock is **risky**.
  - Why place a bet on a non-risky asset?
  - If asset is not risky, everything is known
- The more risky the underlying asset, the more interesting the option.
- Because an option is a one-sided bet (you can't lose more than you paid):

The more risky the underlying stock, the more the option is worth.

Examples: Most active options, 3 October 2006

	A	B	C	D	E	F	G	H	I	J
1	MOST ACTIVE OPTIONS, 3 OCTOBER 2006									
2		Stock								
3	Rank	Symbol	Name	Option expiration	Option exercise price	Put or call?	Stock closing price	Option closing price	Volume	Open Interest
4	1	HAL	Halliburton	20-Apr-07	27.5	Put	26.75	3.025	60,945	7,305
5	2	HAL	Halliburton	19-Jan-07	27.5	Put	26.75	2.450	59,767	61,121
6	3	HAL	Halliburton	19-Jan-07	30.0	Put	26.75	4.100	59,131	68,782
7	4	QQQQ	Nasdaq 100 Index	20-Oct-06	40.0	Put	40.31	0.475	57,073	262,601
8	5	HAL	Halliburton	20-Apr-07	22.5	Put	26.75	1.025	53,901	5,530
9	6	IWM	Russell 2000 Index	17-Nov-06	70.0	Put	71.22	1.650	49,387	203,975
10	7	QQQQ	Nasdaq 100 Index	17-Nov-06	39.0	Put	40.31	0.575	45,666	93,169
11	8	IWM	Russell 2000 Index	20-Oct-06	72.0	Put	71.22	1.650	43,432	154,148
12	9	SPY	S&P Depository Receipts ("Spider")	15-Dec-06	120.0	Put	133.36	0.400	40,125	32,090
13	10	SPY	S&P Depository Receipts ("Spider")	15-Dec-06	139.0	Call	133.36	0.850	40,116	21,036
14	11	SPY	S&P Depository Receipts ("Spider")	15-Dec-06	126.0	Put	133.36	0.925	40,082	32,249
15	12	IWM	Russell 2000 Index	20-Oct-06	71.0	Put	71.22	1.200	39,015	168,035
16	13	QQQQ	Nasdaq 100 Index	20-Oct-06	40.0	Call	40.31	0.875	37,502	196,355
17	14	BMY	Bristol-Myers Squibb	20-Oct-06	22.5	Call	24.82	2.325	31,989	10,934
18	15	SPY	S&P Depository Receipts ("Spider")	20-Oct-06	125.0	Put	133.36	0.075	31,964	25,569
19	16	S	Sprint Nextel	17-Nov-06	17.5	Call	16.97	0.525	31,350	9,078
20	17	IWM	Russell 2000 Index	20-Oct-06	69.0	Put	71.22	0.600	30,777	111,620
21	18	SPY	S&P Depository Receipts ("Spider")	20-Oct-06	137.0	Call	133.36	0.125	30,199	6,261
22	19	QQQQ	Nasdaq 100 Index	20-Oct-06	41.0	Put	40.31	0.975	29,881	94,955
23	20	MMM	MMM	20-Oct-06	85.0	Put	74.03	11.000	25,732	6,364
24										
25	Volume: Number of options traded on 4 Oct. 2006. Each option is for 100 shares, but price quotes are per share.									
26	Open interest: Number of open positions at end of day. A position is closed out (i.e., not open) if the option has been sold by day end.									

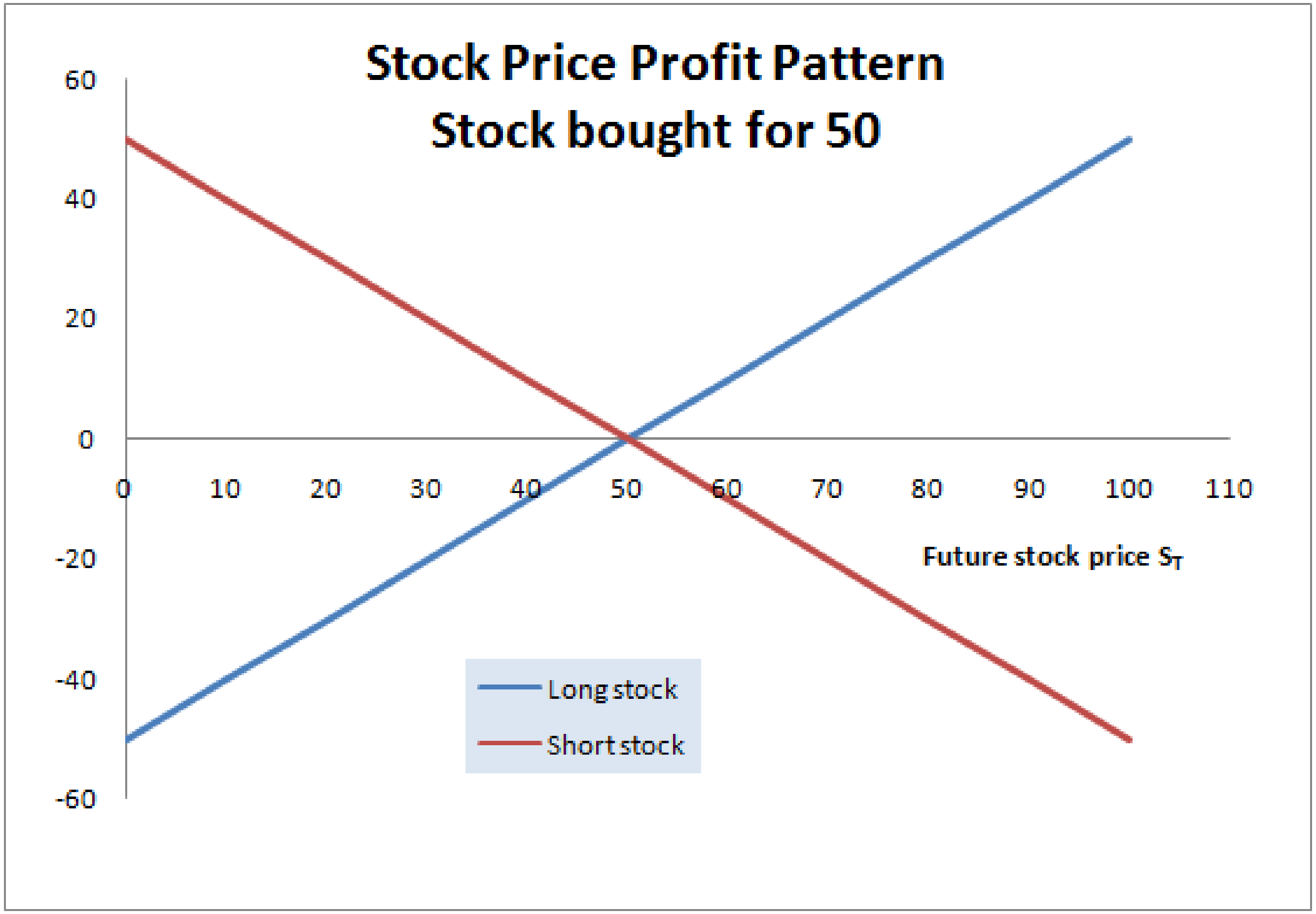
	A	B	C	D	E	F	G	H
1	**HALLIBURTON JAN07 OPTIONS, PRICES ON 3OCT06 Closing price of stock: 26.75, down 1.29**							
2		**Calls**				**Puts**		
3	Exercise price	Closing option price	Volume	Open interest		Closing option price	Volume	Open interest
4	15.0	11.90	20	1,724		0.05	50	1,153
5	17.5	12.70	23	9,449		0.05	140	3,312
6	20.0	7.17	5	13,801		0.20	50	18,277
7	22.5	5.00	339	4,595		0.60	49	15,247
8	25.0	3.20	200	3,306		1.25	423	21,793
9	27.5	1.85	571	3,918		2.50	53,267	61,121
10	30.0	0.95	2,140	12,008		4.10	52,631	68,782
11	32.5	0.55	810	10,527		6.00	216	21,562
12	33.8	0.40	78	2,276		6.90	168	3,378
13	35.0	0.30	231	69,727		8.10	142	20,258
14	36.3	0.20	460	5,989		8.00	2	1,280
15	37.5	0.15	1,249	78,762		9.10	5	5,530
16	40.0	0.10	32	85,037		11.01	16	322
17	42.5	0.05	170	24,439		15.70	135	1,920
18	45.0	0.05	6	8,509		17.10	76	0
19	47.5	0.05	10	6,591		14.60	76	0
20	50.0	0.05	20	10,528		17.00	52	0
21	55.0	0.05	12	694		22.10	28	0

Most volume is concentrated around at-the-money options

Option profit patterns

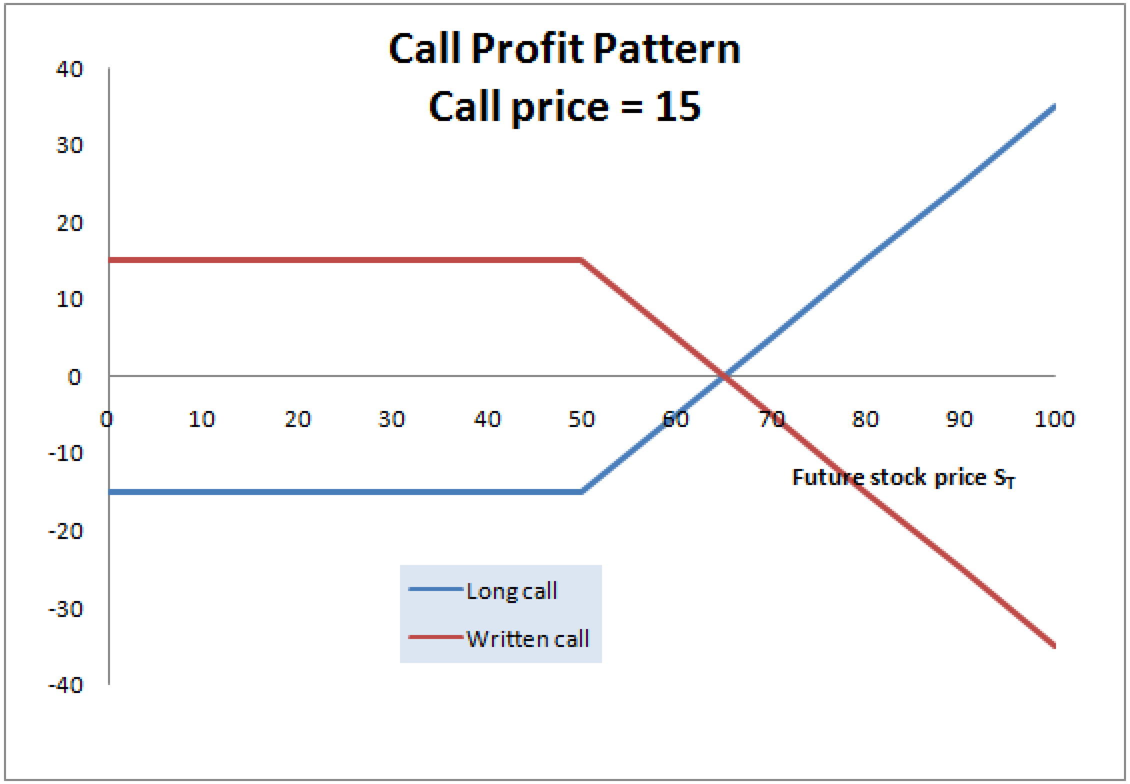
- Popular sport! Graph payoffs for call option, put option, stock at exercise date T as function of stock price S_T
- Graph payoffs of **combinations**
  - Stock + put ("protective put")
  - Two calls with different exercise prices ("spread")
  - Three calls or puts with different exercise prices ("butterfly)

Stock profit pattern



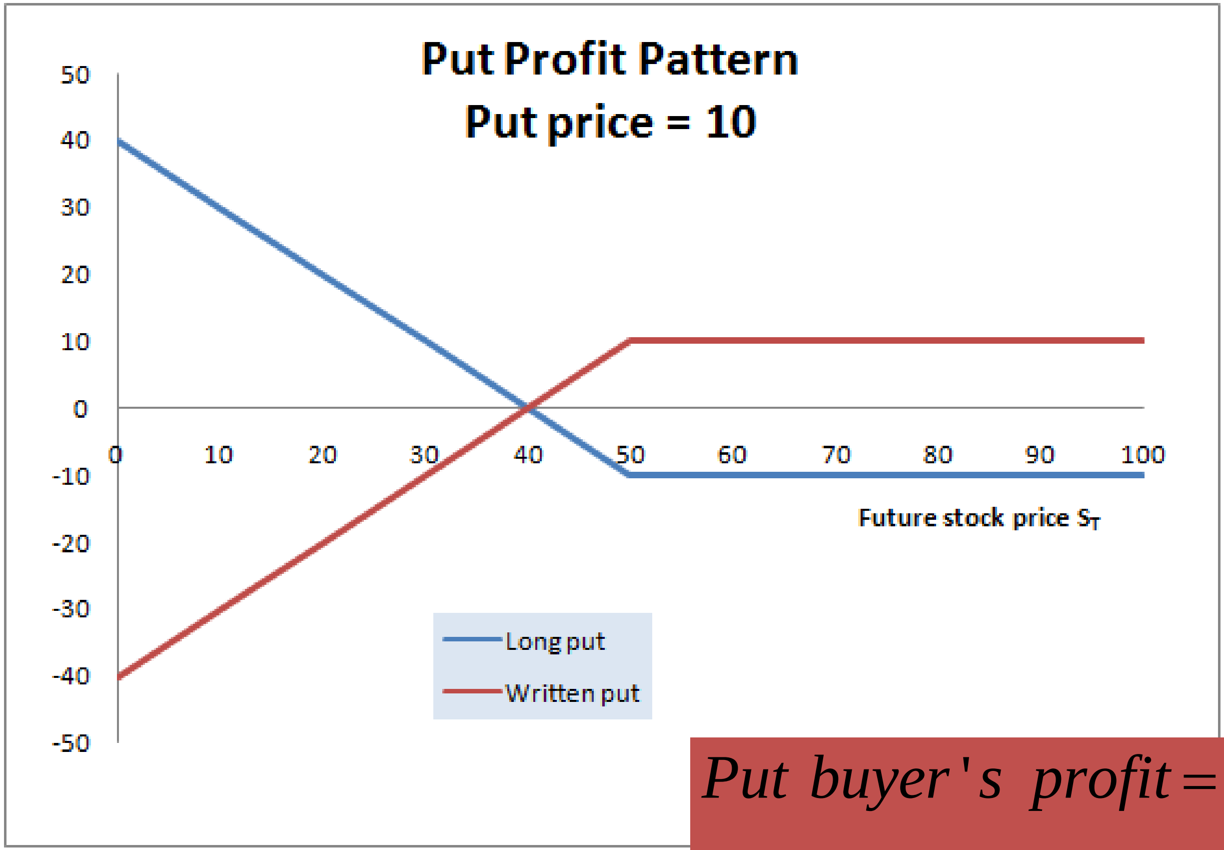
- ❑Make money on bought stock if price rises
- ❑Make money on shorted stock if price falls

Call option profit pattern



$$
\begin{aligned}
\textit{Call buyer's profit} &= \mathrm{Max}(S_T - X, 0) - C_0 \\
&= \mathrm{Max}(S_T - 50, 0) - 15 \\
&= \begin{cases} -15 & \textit{if } S_T \leq 50 \\ S_T - 65 & \textit{if } S_T > 50 \end{cases}
\end{aligned}
$$

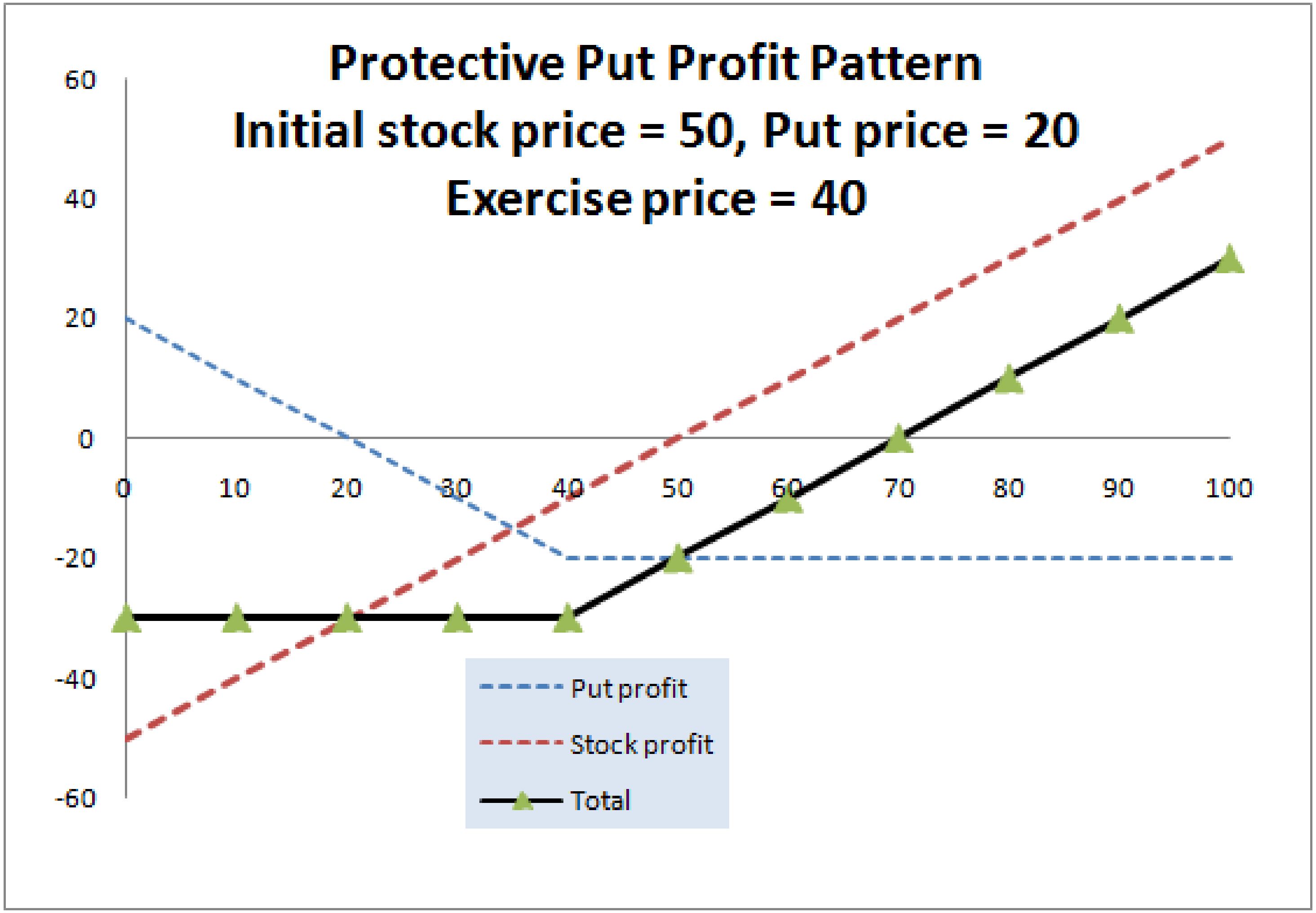
Put option profit pattern



$$
\begin{aligned}
Put\ buyer's\ profit &= \mathrm{Max}(0, X - S_T) - P_0 \\
&= \mathrm{Max}(0, 50 - S_T) - 10 \\
&= \begin{cases} -40 + S_T & if\ \ S_T \leq 50 \\ -1 & if\ \ S_T > 50 \end{cases}
\end{aligned}
$$

Protective put

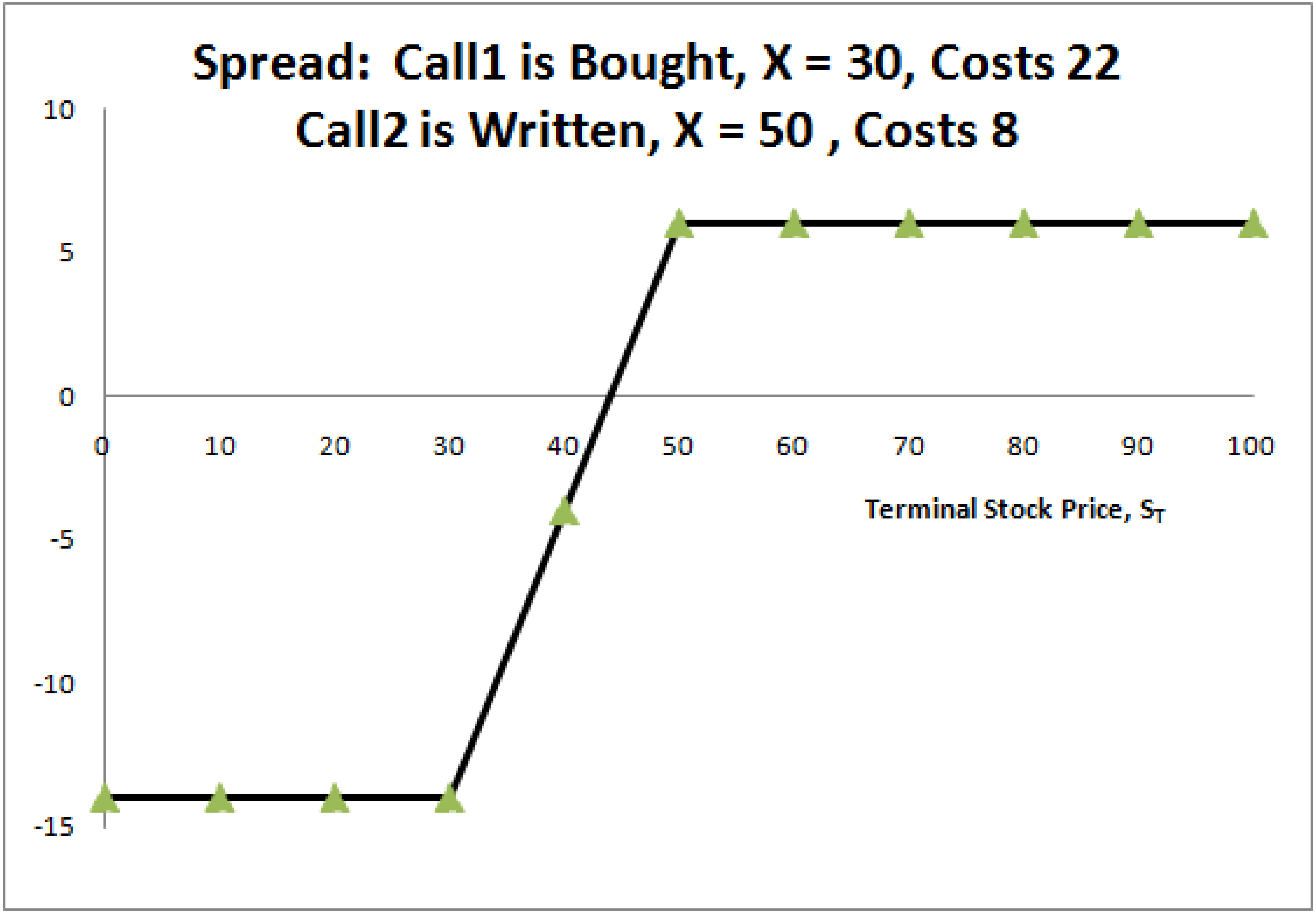
- Buy stock today at price S_0
- But a put for P_0 with exercise price X
- Cost today:
  - S_0+P_0
- Payoff at time T:
  - $S_T+\mathrm{Max}(X-S_T,0)$



Protective put payoff pattern looks like that of a call. Is it true that

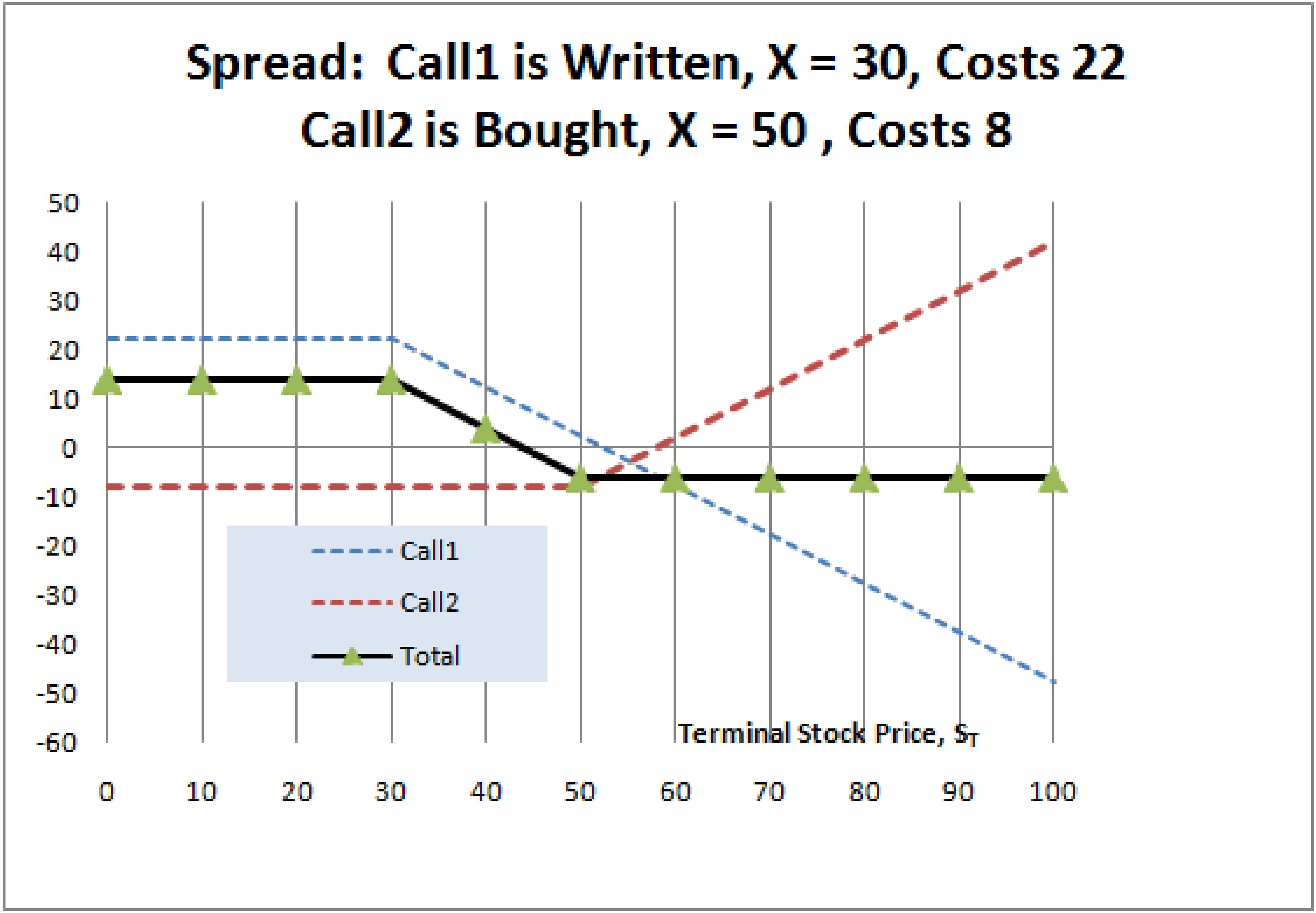
❑ Stock + Put = Call

❑ Almost but not quite—see “Put-Call Parity” (later on)


Spread: Call1 is Bought, X = 30, Costs 22
Call2 is Written, X = 50 , Costs 8
10
5
0
-5
-10
-15
0
10
20
30
40
50
60
70
80
90
100
Terminal Stock Price, S_T

Call spread

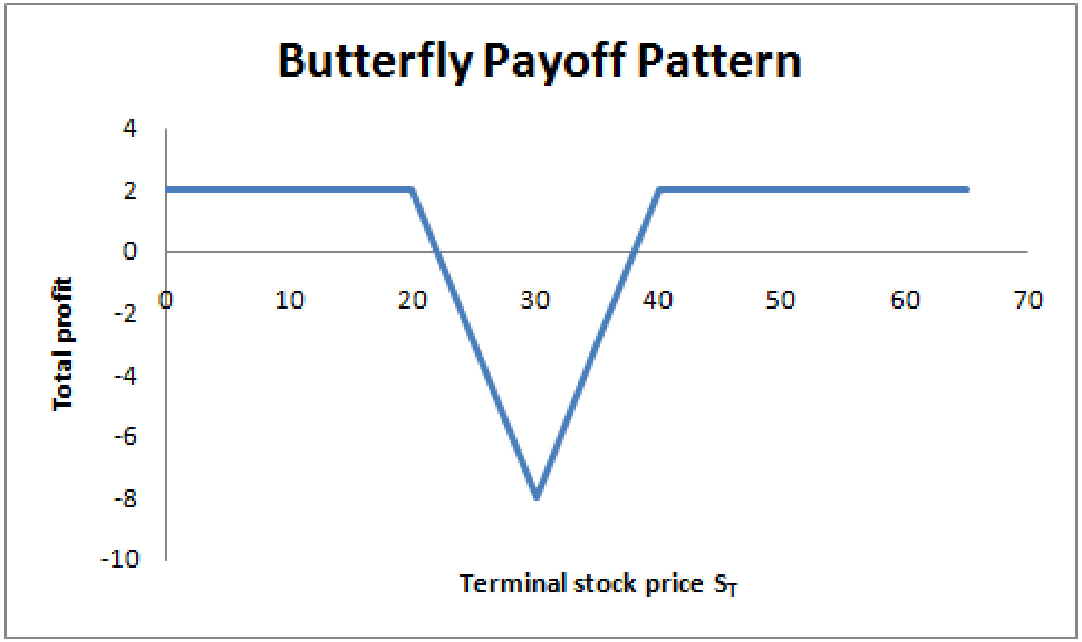
- Buy one call with exercise price X_{high}
- Write call with exercise price X_{low}
- Both with same time T to maturity
- Profit at T:
  - $-C(X_{high})+C(X_{low}) + Max(S_T - X_{high},0)-Max(S_T - X_{low},0)$


Spread: Call1 is Written, X = 30, Costs 22
Call2 is Bought, X = 50 , Costs 8
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
0
10
20
30
40
50
60
70
80
90
100
Call1
Call2
Total
Terminal Stock Price, S_T

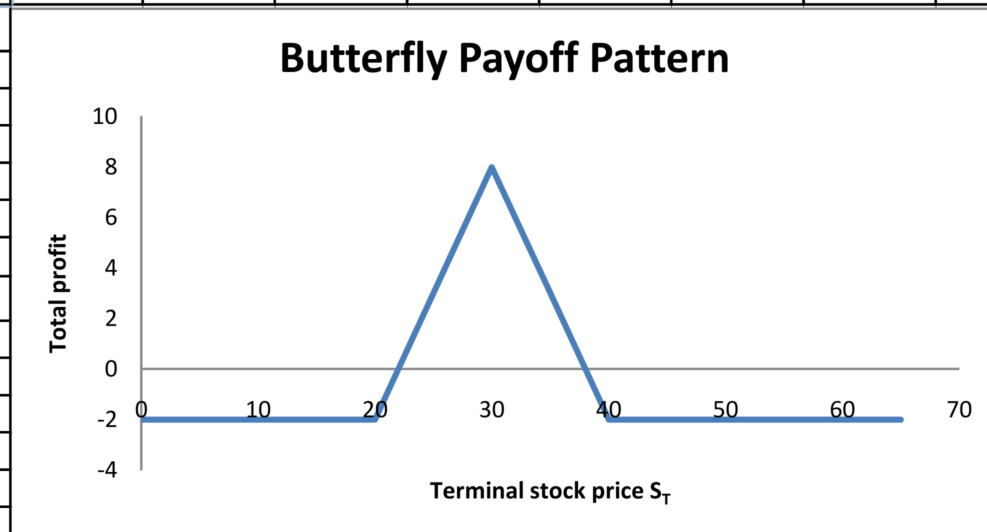
Butterfly

- Combination of three puts or calls with different exercise prices (some long, some short)
- Total number of positions (short + long) adds to zerio

	A	B	C	D	E
1	**BUTTERFLY WITH THREE CALLS**				
2		**Call1**	**Call2**	**Call3**	
3	Exercise price	20	30	40	
4	Call price	10	6	4	
5	Number of calls purchased	-1	2	-1	
6					
7	**Payoff example**				
8	S_T, terminal stock price	50			
9	Profit, Call1	-20	<-- =B5*(MAX(B8-B3,0)-B4)		
10	Profit, Call1	28	<-- =C5*(MAX(B8-C3,0)-C4)		
11	Profit, Call1	-6	<-- =D5*(MAX(B8-D3,0)-D4)		
12	Total	2	<-- =SUM(B9:B11)		



	A	B	C	D	E
1	**SWITCH THE POSITIONS**				
2		**Call1**	**Call2**	**Call3**	
3	Exercise price	20	30	40	
4	Call price	10	6	4	
5	Number of calls purchased	1	-2	1	
6					
7	**Payoff example**				
8	S_T, terminal stock price	50			
9	Profit, Call1	20	<-- =B5*(MAX(B8-B3,0)-B4)		
10	Profit, Call1	-28	<-- =C5*(MAX(B8-C3,0)-C4)		
11	Profit, Call1	6	<-- =D5*(MAX(B8-D3,0)-D4)		
12	Total	-2	<-- =SUM(B9:B11)		



1 call bought with exercise price 20
2 call written with exercise price 30
1 call bought with exercise price 40

Option arbitrage propositions

- Facts about option prices
- Derived without much/any assumptions about stochastic process of stock price
- Derived only from definitions

Arbitrage position 0

- Consider an American call costing C_0, with exercise price X, where the stock price is S_0. Then

  C_0 must be > $Max(S_0\text{-}X,0)$

- Proof by example: Suppose $C_0 = 5$, $S_0 = 50$, and X = 40.
- **Make immediate profit**
  - Buy call: - 5
  - Exercise immediately: -40
  - Sell stock immediately: +50

Arbitrage proposition 1

- Consider a European call costing C_0, with exercise price X, where the stock price is S_0. Then

  C_0 must be > $Max(S_0\text{-}PV(X),0)$

- Proposition 0 is trivial
- Proposition 1 is deep (proof in book)

Arbitrage proposition 2

- It is never optimal to early-exercise an American call written on a stock which doesn't pay dividends before the option maturity T.
- Another interpretation: If you're thinking about early-exercising a call:
  - SELL THE CALL, don't exercise it
  - You'll make more money

Proof by example of Prop. 2

- You own a call with 0.5 years to maturity.
  - Call exercise, X=50
  - Current stock price, S=80
  - Interest rate, r = 6%
- Immediate early exercise:
  Payoff = S – X = 30
- By Prop. 1, market price of call is at least
  Max(S – PV(X),0)=80 – ($\exp^{-0.5*6\%}$)50
  = 80-0.97*50=31.45
- Better off selling the call than exercising

Conclusion from Prop. 2

- The American feature of calls is often worthless
- In many cases: American call and European call have same value
- Not true for puts: European put worth less than American put

Proposition 4: Put-call parity

- Consider a European put and call on the same stock. Put and call have same exercise price X. Stock pays no dividends before option exercise date T.
- Then:

$$\underbrace{P_0 + S_0}_{\substack{-\\ \text{Price of put} \\ \text{plus stock}}} = \underbrace{C_0 + PV\left(X\right)}_{\substack{-\\ \text{Price of call} \\ \text{plus present} \\ \text{value of exercise}}}$$

Proposition 5: Call price convexity

- Consider three calls on same stock with same maturity T. Assume that Call1 has exercise price X_1, Call2 has X_2, Call3 has X_3.
- Assume $X_1 < X_2 < X_3$ and equally spaced: $X_2 = (X_1+X_3)/2$.
- Then

$$Call2 < \frac{Call1 + Call3}{2}$$

Proposition 5 example and counterexample

- Use butterfly spread previously illustrated
- When Proposition 5 condition is violated, there is an arbitrage opportunity.
- Violation of Prop. 5:

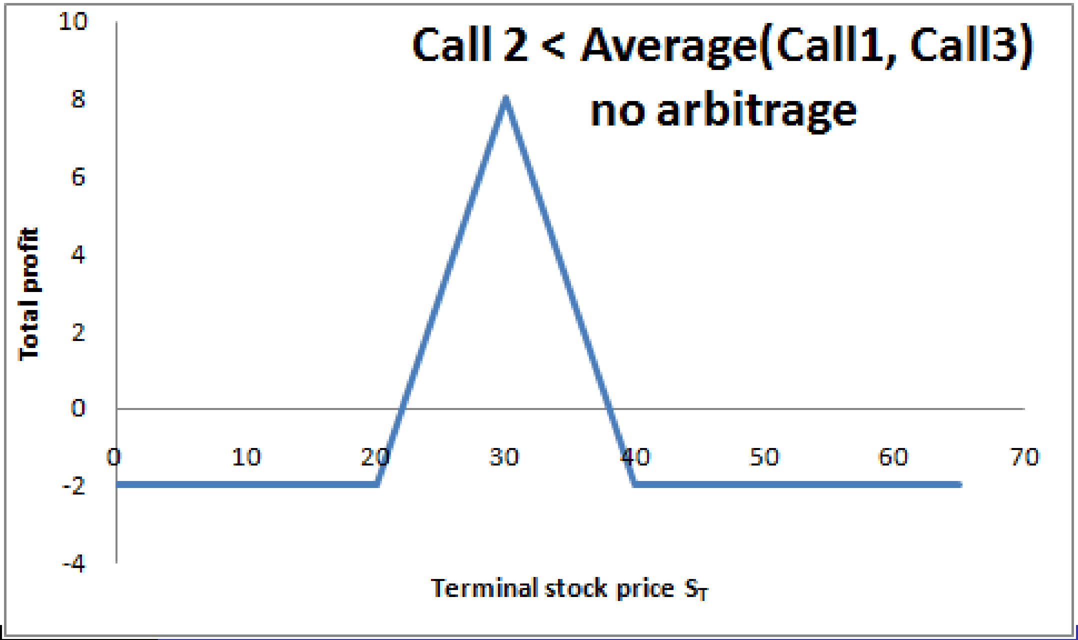
$$Call2 > \frac{Call1 + Call3}{2}$$

No arbitrage:

$$Call2 < \frac{Call1 + Call3}{2}$$

	A	B	C	D	E
1	**PROPOSITION 5 CONDITION HOLDS**				
2		**Call1**	**Call2**	**Call3**	
3	Exercise price	20	30	40	
4	Call price	10	6	4	
5	Number of calls purchased	1	-2	1	

No arbitrage: Sometimes you win, sometimes you lose.



Arbitrage:

$$Call2 >= \frac{Call1 + Call3}{2}$$

	A	B	C	D
1	**PROPOSITION 5 CONDITION VIOLATED**			
2		**Call1**	**Call2**	**Call3**
3	Exercise price	20	30	40
4	Call price	10	8	4
5	Number of calls purchased	1	-2	1

ARBITRAGE:
You NEVER lose!